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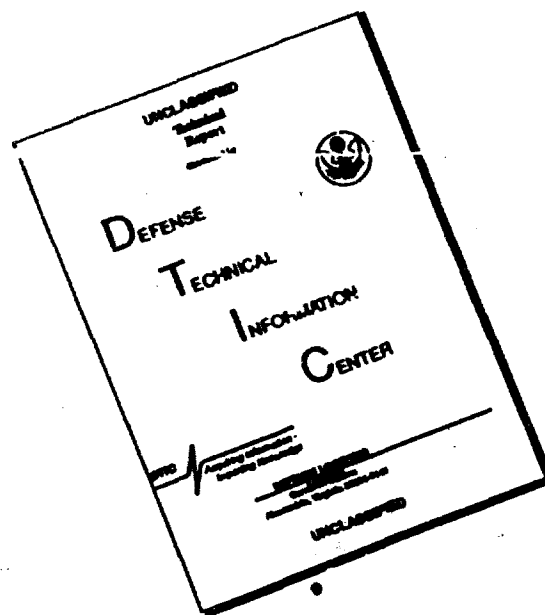
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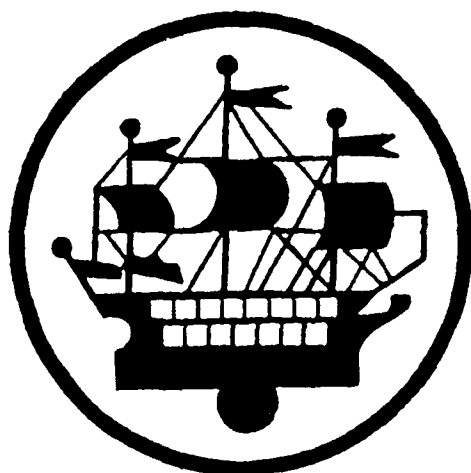
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BOTTOM SEDIMENT EFFECTS IN A NUMERICAL PARABOLIC EQUATION ENSEMBLE AVERAGE PROPAGATION

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ABSTRACT

A method of numerically computing average pressure for an ensemble has been developed using a normal mode analysis of the parabolic equation [1]. It has been used to compute the transmission loss and phase for the average solutions in the continuous wave case where the vertical variation is smoothly varying. In this paper an alternative choice of basis functions allows one to consider sound velocity profiles with discontinuities while preserving numerical convergence. These discontinuities arise when the sound velocity profile makes a rapid jump between layers of differing speeds. Such a difference occurs between the water column and higher velocity fluid bottoms which model sediment layers. Calculations using energy loss due to sediment penetration allow better estimates of attenuation.

INTRODUCTION

The discussion below modifies a numerical normal method for the propagation of the average of the full complex field in [1]. The method propagates the complex pressure field in terms of a vertical transform (similar to a discrete sine transform) corresponding to eigenfunctions of a piecewise constant sound velocity profile. First, the transform is used to re-express equations in the coefficient space of a reference eigenfunction series. Because this change from the piecewise constant function to the reference function involves continuous differen-

tiable functions, the convergence of series will be at least $\mathcal{O}(1/n^2)$. Environmental fluctuation of sound speed changes the eigenfunctions, and these changes can be expressed as orthogonal matrices. Numerical matrix routines [3] quantify the changes and permit their evaluation, as well as eigenvalue changes. Finally, the numerical algorithm computes average solutions, taking into account the variation due to environmental fluctuation.

The generalized parabolic equation corresponds to the rightward propagating solutions of the Helmholtz equation [2]:

$$\left(i\partial_r + \sqrt{\partial_z^2 + q + k^2}\right)u = 0. \tag{1}$$

The index of refraction $\frac{c_0}{c(z)} = n \approx 1$, and the sound speed term has jump discontinuities between layers of differing speed:

$$q = q(z) = \hat{q}(z) + q_{\epsilon=0}(z) + q_\epsilon(z) = k^2(n^2(z) - 1), \tag{2}$$

where $\hat{q}(z)$ is piecewise constant, $q_{\epsilon=0}$ and q_ϵ are continuous and differentiable. For simplicity, the parameter $0 < \epsilon < 1$. The function q is depth dependent and range constant. The vertical and horizontal equations are, respectively

$$(\partial_z^2 + q + \lambda_n)\xi_n = 0 \tag{3}$$

$$\left(i\partial_r + \sqrt{k^2 - \lambda_n}\right)\psi_n = 0. \tag{4}$$

REFERENCE VERTICAL NORMAL MODES

This section introduces a vertical sine-like transform of (3) based on $\hat{q}$. Here we consider pressure release top and bottom surfaces. Collins has already used similar boundary conditions with success in FEPE [4]. A single fluid basement of higher velocity models sediment, although several layers may be used. (The analysis for a semi-infinite fluid bottom is nearly identical for the eigenvalues, but in the continuous spectrum, it involves integro-differential equations with Hilbert transforms.)

Since we consider only two layers the $\hat{q}$ profile is

$$\hat{q} = \begin{cases} q_0, & 0 < z < b \\ q_1, & b < z < h \end{cases} \tag{5}$$

with $\hat{q} = q_0 - q_1 > 0$ since the sediment sound speeds are higher. There are case (a) modes confined to the top fluid and (b) modes which have significant

sediment penetration.

(a) Top confined eigenvalues have

$$-q_0 < \mu_n < -q_1.$$

Use $\mu_n = \kappa_n^2 - q_0 = -q_1 - \nu_n^2$ to define κ_n, the upper layer form of the eigenvalue, and ν_n, the lower layer form. Set $\tilde{z} = h - z$. The eigenfunction and its derivative are

$$\phi_n = \begin{cases} s_0 \sin \kappa_n z, & z < b \\ s_1 \sinh \nu_n \tilde{z}, & \tilde{z} < \tilde{b} \end{cases} \qquad \partial_z \phi_n = \begin{cases} s_0 \kappa_n \cos \kappa_n z, & z < b \\ -s_1 \nu_n \cosh \nu_n \tilde{z}, & \tilde{z} < \tilde{b} \end{cases}$$

Continuity dictates matching at layer interface $z = b$, $\tilde{z} = \tilde{b}$. Their ratio is $\frac{\tan \kappa_n b}{\kappa_n} = -\frac{\tanh \nu_n \tilde{b}}{\nu_n}$ .Note that as $\nu \downarrow 0$, $\frac{\tanh \nu_n \tilde{b}}{\nu_n} \uparrow \tilde{b}$; so $1 \leq n < \frac{\sqrt{q}}{\pi} + \frac{1}{2}$, unless $\tan b\sqrt{q} < -\tilde{b}\sqrt{q}$ in which case $n < \frac{\sqrt{q}}{\pi}$. Determine the eigenvalues numerically by solving

$$\kappa_n = \frac{1}{b}\left(n\pi - \mathrm{Tan}^{-1}\frac{\kappa_n}{\nu_n}\tanh \nu_n\tilde{b}\right).$$

Normalize (where constant density has been used) $1 = \|\phi_n\|_2^2 = \int_0^h \phi_n^2(z)dz$ to find

$$s_0 = \left[\frac{b\sinh^2 \nu_n\tilde{b} - \tilde{b}\sin^2 \kappa_n b}{2\sinh^2 \nu_n\tilde{b}} - \frac{\tilde{q}\sin 2\kappa_n b}{4\kappa_n\nu_n^2}\right]^{-1/2}.$$

(b) For sediment penetrating eigenvalues $\hat{q} + \mu \geq 0$ everywhere. Thus, the top and bottom forms of the eigenvalues are defined by $\mu_n = \kappa_n^2 - q_0 = \nu_n^2 - q_1$. As before, the eigenfunction and derivative match at layer interface to give ratio $\frac{\kappa_n}{\nu_n} = -\frac{\tan \kappa_n b}{\tan \nu_n \tilde{b}}$. Set $\check{b} = \tilde{b} - b$. To find eigenvalues numerically, solve

$$\nu_n = \frac{\zeta}{h} - \frac{b\tilde{q}}{\zeta}\left[1 + \sqrt{1 + (h\check{b}\tilde{q})/\zeta^2}\right]^{-1},$$

where $\zeta = n\pi + (-1)^n \mathrm{Sin}^{-1}\left[\frac{\kappa_n - \nu_n}{\kappa_n}\sin \kappa_n b \cos \nu_n \tilde{b}\right]$. This requires $n \geq \frac{b\sqrt{q}}{\pi}$ and $n^2 \geq -\frac{(h\tilde{q})\check{b}}{\pi^2}$. Normalization implies $s_0 = \left[\frac{b}{2} + \frac{\check{b}\sin^2 \kappa_n b}{2\sin^2 \nu_n \tilde{b}} + \frac{\tilde{q}\sin 2\kappa_n b}{4\kappa_n\nu_n^2}\right]^{-1/2}$. Finally, the ϕ_n and μ_n depend upon frequency through the k^2 term in $\hat{q}$.

Consider the ϵ independent case of $q = \hat{q}(z) + q_0(z)$ in equation (3) . It has eigenfunctions $\psi_n(z)$ and eigenvalues λ_n. Expand these vertical eigenfunctions in the $\hat{q}$ eigenfunction series $\psi_n = \sum_{m=1} \phi_m R_{mn}$. This allows the expression of the differential equation in the series

$$0 = \sum_{m=1} \phi_m R_{mn}[-\mu_m + (\lambda_n + q_0)].$$

Define the inner product of velocity and $\hat{q}$ eigenfunctions

$$(q_0\phi_k, \phi_m) \equiv \int_{-h}^{0} q_0(z)\phi_k(z)\phi_m(z)\,dz\,.$$

Reformulation of the original equation (1) requires definitions of the several vectors and matrices: $\mathbf{R}_{mk} = (\psi_k,\ \phi_m)$ is the matrix containing the $\hat{q}$ eigenfunction transform of ψ_k eigenfunctions; the sound velocity profile information is incorporated into the matrix $\mathbf{q}_0$ with component $q_{0,mn} = (q_0\phi_m, \phi_n)$. These now satisfy the eigenvalue equation

$$\Lambda = \mathbf{R}^T(\mathbf{M} - \mathbf{q}_0)\,\mathbf{R},\ \ \Lambda = \mathrm{diag}(\lambda_1,\ \lambda_2,\ \ldots), \tag{6}$$

where $\mathbf{M} = \mathrm{diag}(\mu_1,\ \mu_2,\ \ldots)$ is the diagonal matrix of $\hat{q}$ eigenvalues.

VERTICAL MODES IN THE ENSEMBLE

Next take an ensemble velocity profile expressed in terms of the single parameter ϵ in $q = \hat{q} + q_0 + q_\epsilon$. The ϵ-dependent eigenfunctions $\psi_{\epsilon\,m}$ of (3) may be written in ψ_m series:

$$\psi_{\epsilon\,n} = \sum_{m=1}^{\infty} \psi_m P_{\epsilon\,mn}, \tag{7}$$

where $P_{\epsilon\,mn} = (\psi_m,\ \psi_{\epsilon n})$. Computation of the matrix diagonalization gives

$$\mathbf{P}_\epsilon^T(-\Lambda + \mathbf{R}^T \mathbf{q}_\epsilon \mathbf{R})\mathbf{P}_\epsilon = -\Lambda_\epsilon. \tag{8}$$

A single element of the ensemble (ϵ fixed) is a solution to

$$\left(i\partial_r + \sqrt{\partial_z^2 + \hat{q} + q_0 + q_\epsilon}\right) u_\epsilon = 0. \tag{9}$$

Express the solution u_ϵ of (9) in an eigenfunction series of the $\psi_{\epsilon\,m}$ corresponding to $\hat{q} + q_0 + q_\epsilon$ in (3):

$$u_\epsilon = \sum_{m=1}^{\infty} \psi_{\epsilon\,m} w_{\epsilon\,m}.$$

The coefficient vectors for the eigenfunction series are related by $\mathbf{R}\mathbf{P}_\epsilon \mathbf{w}_\epsilon = \mathbf{u}_\epsilon$. Eq. (9) means vectors corresponding to the ϵ parameterized eigenfunctions satisfy the vector differential equation

$$\partial_r \mathbf{w}_\epsilon = i\sqrt{k^2 - \Lambda_\epsilon}\,\mathbf{w}_\epsilon, \tag{10}$$

where $\sqrt{k^2 - \Lambda_\epsilon}$ is the diagonal matrix of horizontal eigenvalues.

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AVERAGE SOLUTION TO THE PARABOLIC EQUATION

Linear algebra shows that $P_\epsilon = e^{A(\epsilon)}$ where $A(\epsilon)$ is anti-hermitian [5]. Under suitable conditions, $A(\epsilon) = \epsilon A$. In this case

$$\mathbf{P}_\epsilon = e^{\epsilon\,\mathbf{A}} = \mathbf{U}\exp(\epsilon\,i\Theta)\mathbf{U}^*, \tag{11}$$

where $\exp(\epsilon\,i\Theta) = \mathrm{diag}(e^{\epsilon\,i\theta_1}, e^{\epsilon\,i\theta_2}, \ldots)$ and $\mathbf{U}$ is a constant unitary matrix [6]. Eq. (10) has a solution vector

$$\mathbf{w}_\epsilon = \exp(i\sqrt{k^2 - \Lambda_\epsilon}[r - r_0])\mathbf{w}(r_0).$$

This is orthogonally transformed back to the $\hat{q}$ eigenfunctions transformed coefficients u_ϵ. These are averaged with respect to ϵ to give the ensemble averaged with respect to ϵ

$$\langle u_\epsilon \rangle = \mathbf{R}\left\langle \mathbf{P}_\epsilon \exp(i\sqrt{k^2 - \Lambda_\epsilon}[r - r_0])\mathbf{P}_\epsilon^T \right\rangle \mathbf{R}^T \mathbf{u}(r_0).$$

The parabolic equation (1) has an ensemble averaged solution expressed in terms of the $\hat{q}$ eigenfunction coefficient vectors

$$\langle \mathbf{u}_\epsilon \rangle = (\mathbf{RU})\,\langle\Gamma\rangle\,(\mathbf{RU})^*\mathbf{u}(r_0), \tag{12}$$

where

$$\Gamma = e^{\epsilon\,i\Theta}\mathbf{U}^* e^{i\sqrt{k^2 - \Lambda_\epsilon}[r - r_0]}\mathbf{U}e^{-\epsilon\,i\Theta}$$

The average coupling matrix $\langle\Gamma\rangle$ has components

$$\langle\Gamma_{mn}\rangle = \sum_{j=1}^{N} \bar{U}_{jm}U_{jn}\,\langle\gamma\rangle_{mnj}$$

with

$$\langle\gamma\rangle_{mnj} = \int_{-\infty}^{\infty} \exp\left(\epsilon\,i[\theta_m - \theta_n] + i\sqrt{k^2 - \lambda_{\epsilon,j}}[r - r_0]\right)dp(\epsilon),$$

where $dp(\epsilon) = \frac{dp}{d\epsilon}d\epsilon$ is the probability density.

ATTENUATION ESTIMATES

In general, if the depth dependent attenuation coefficient is $\vartheta(z)$, then the modal attenuation $e^{-\delta_{\epsilon n}[r - r_0]}$ is given by

$$\delta_{\epsilon n} = \int_0^h \frac{\vartheta(z)}{K_{\epsilon n}}\psi_{\epsilon n}^2(z)dz \tag{13}$$

where $K_{\epsilon n} = \sqrt{k^2 - \lambda_\epsilon}$ [7]. For comparison, ϵ in [7] is related to $\beta(z)$ above by inclusion of a factor of $\frac{\omega}{c}$ in β. Computationally, one may rewrite (13) using (7) and $\mathbf{R}$ to find

$$\delta_{\epsilon n} = \frac{1}{K_{\epsilon n}} \sum_{kjj'k'} R_{jk} P_{\epsilon kn} R_{j'k'} P_{\epsilon k'n} B_{jj'}$$

where $B_{jj'} = \int_0^h \beta(z)\phi_j(z)\phi_{j'}(z)dz$. If, for example, β is constant in the sediment and vanishes in the water column, and $k \neq k'$ are in the non-penetrating spectrum, then $B_{jj'} = \beta s_j s_{j'} \left(\frac{\sinh(\nu_j + \nu_{j'})\tilde{z}}{\nu_j + \nu_{j'}} - \frac{\sinh(\nu_j - \nu_{j'})\tilde{z}}{\nu_j - \nu_{j'}} \right)$. Finally, if we wish to find the ensemble averaged attenuation $\delta_{\epsilon n}$,

$$\langle \delta_{\epsilon n} \rangle = \sum_{kjj'k'} R_{jk} \Pi_{k k' n} R_{j'k'} B_{jj'}$$

where $\Pi_{k k' n} = \sum_{jj'} U_{kj} \bar{U}_{nj} U_{k'j'} \bar{U}_{nj'} \int \frac{1}{K_{\epsilon n}} e^{\epsilon i(\theta_j + \theta_{j'})} dp(\epsilon)$.

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